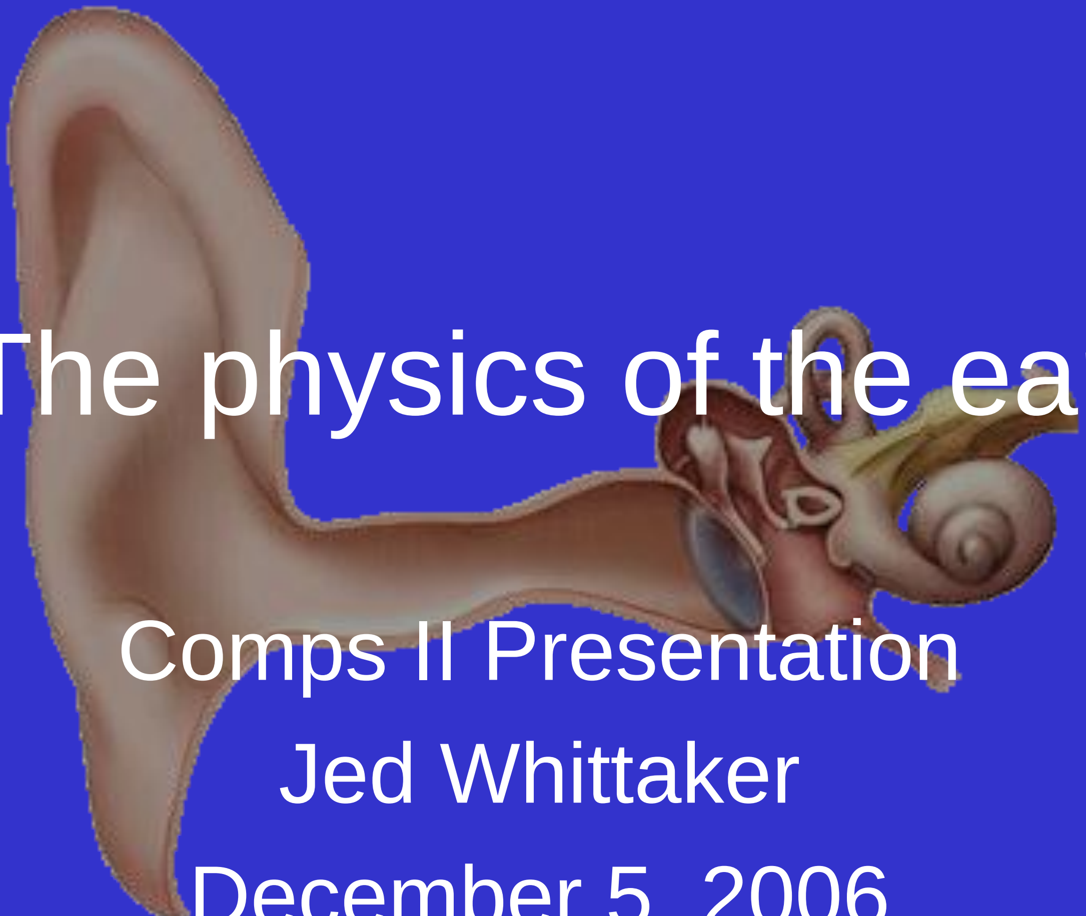


The physics of the ear

Comps II Presentation

Jed Whittaker

December 5, 2006

Outline

I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
  a. Anatomy
  b. Theories of cochlear function
  c. Traveling wave theory
VII. Current work
  a. Physical model
  b. Mathematical model

Outline

I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
  a. Anatomy
  b. Theories of cochlear function
  c. Traveling wave theory
VII. Current work
  a. Physical model
  b. Mathematical model

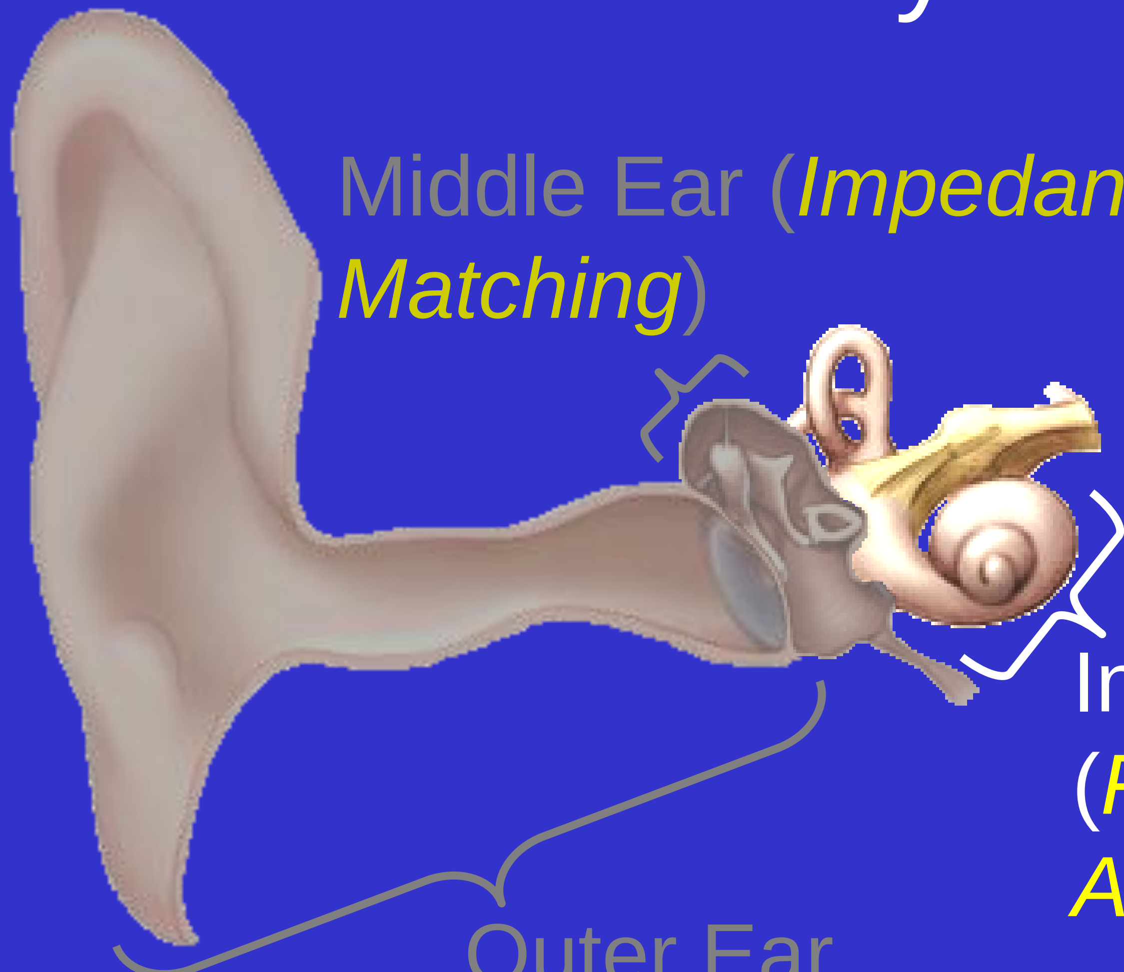
Ear anatomy

http://www.nidcd.nih.gov/StaticResources/health/hearing/images/normal_ear.asp

Ear anatomy

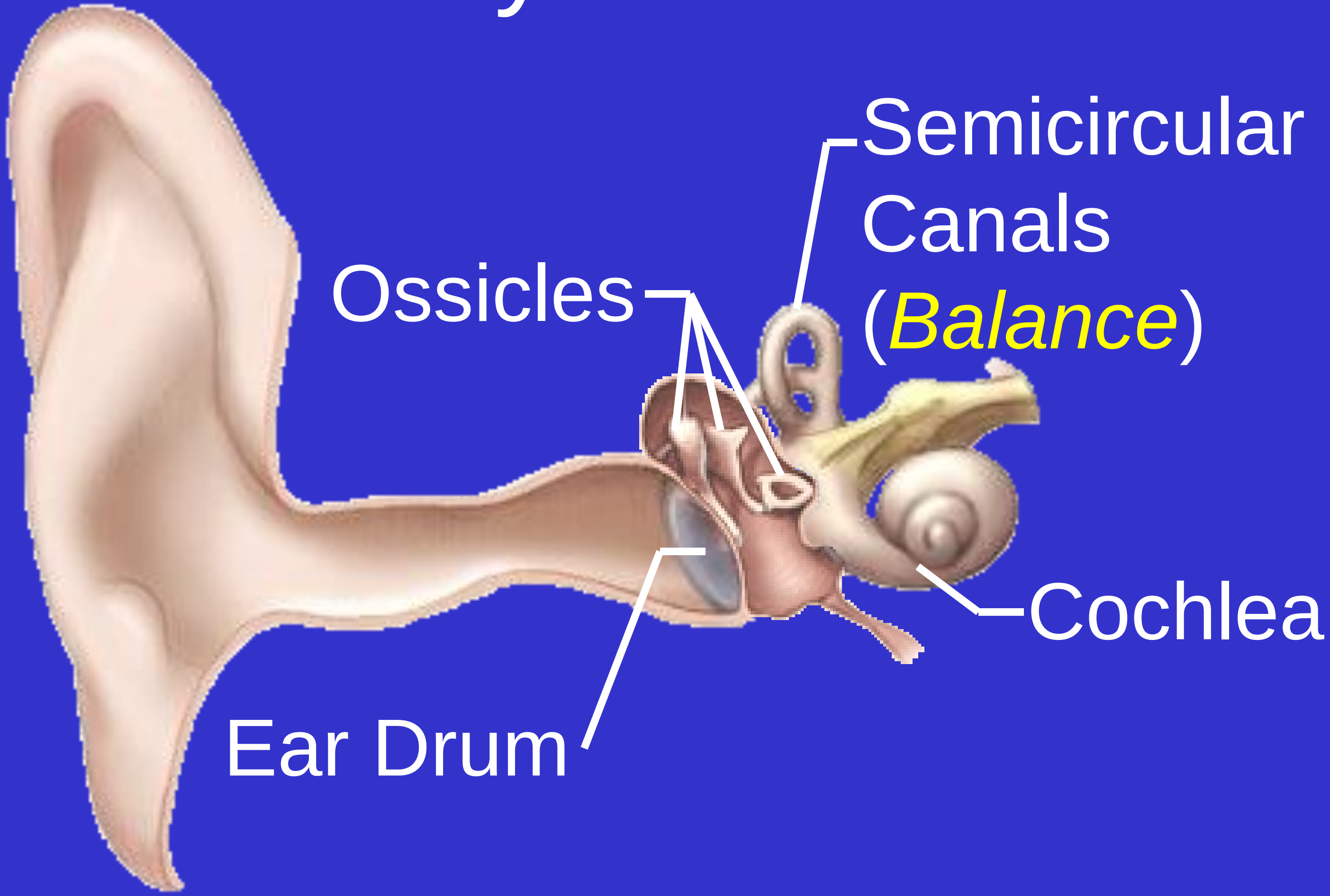
http://www.nidcd.nih.gov/StaticResources/health/hearing/images/normal_ear.asp

Ear anatomy



http://www.nidcd.nih.gov/StaticResources/health/hearing/images/normal_ear.asp

Ear anatomy substructures



Outline

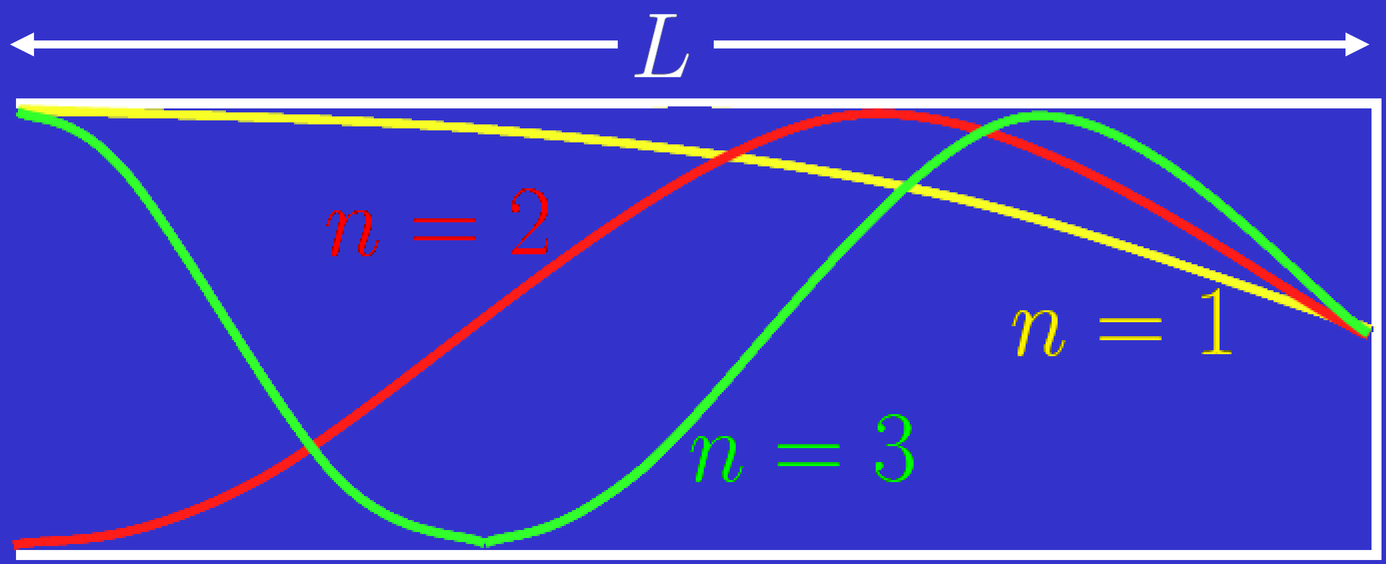
I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
  a. Anatomy
  b. Theories of cochlear function
  c. Traveling wave theory
VII. Current work
  a. Physical model
  b. Mathematical model

The outer ear

The human ear is most responsive at about 3,000 Hz

Most speech occurs at about 3,000 Hz

Partially closed pipe resonator model



$$F_n = \frac{nc}{4L}$$

frequency → F

mode → n

speed of sound → c

Outer ear resonator

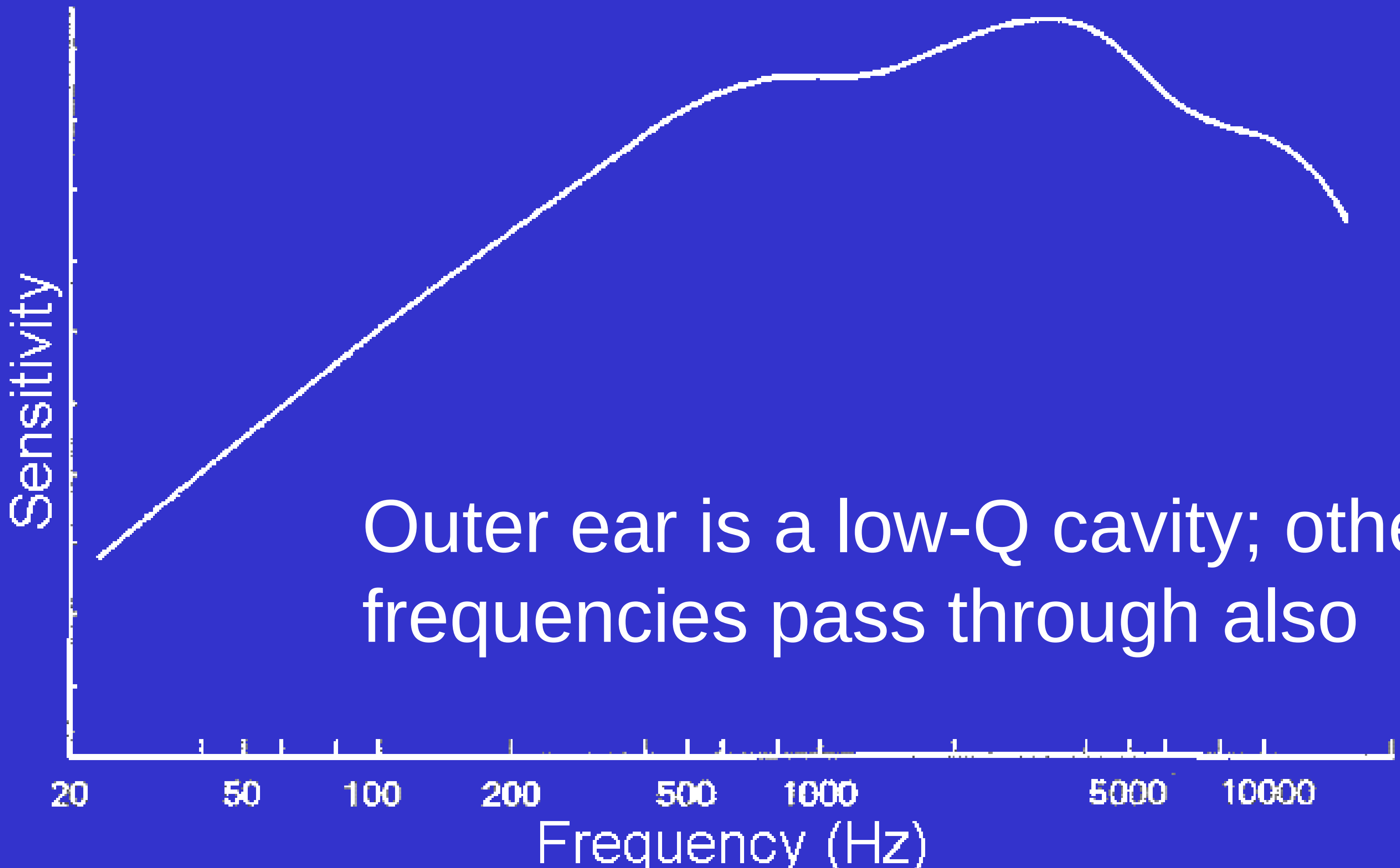
The length of the human auditory canal is

$$L \sim 28mm$$

This gives a fundamental mode of

$$F_1 \sim 3000Hz$$

Outer ear Q



Outer ear is a low-Q cavity; other frequencies pass through also

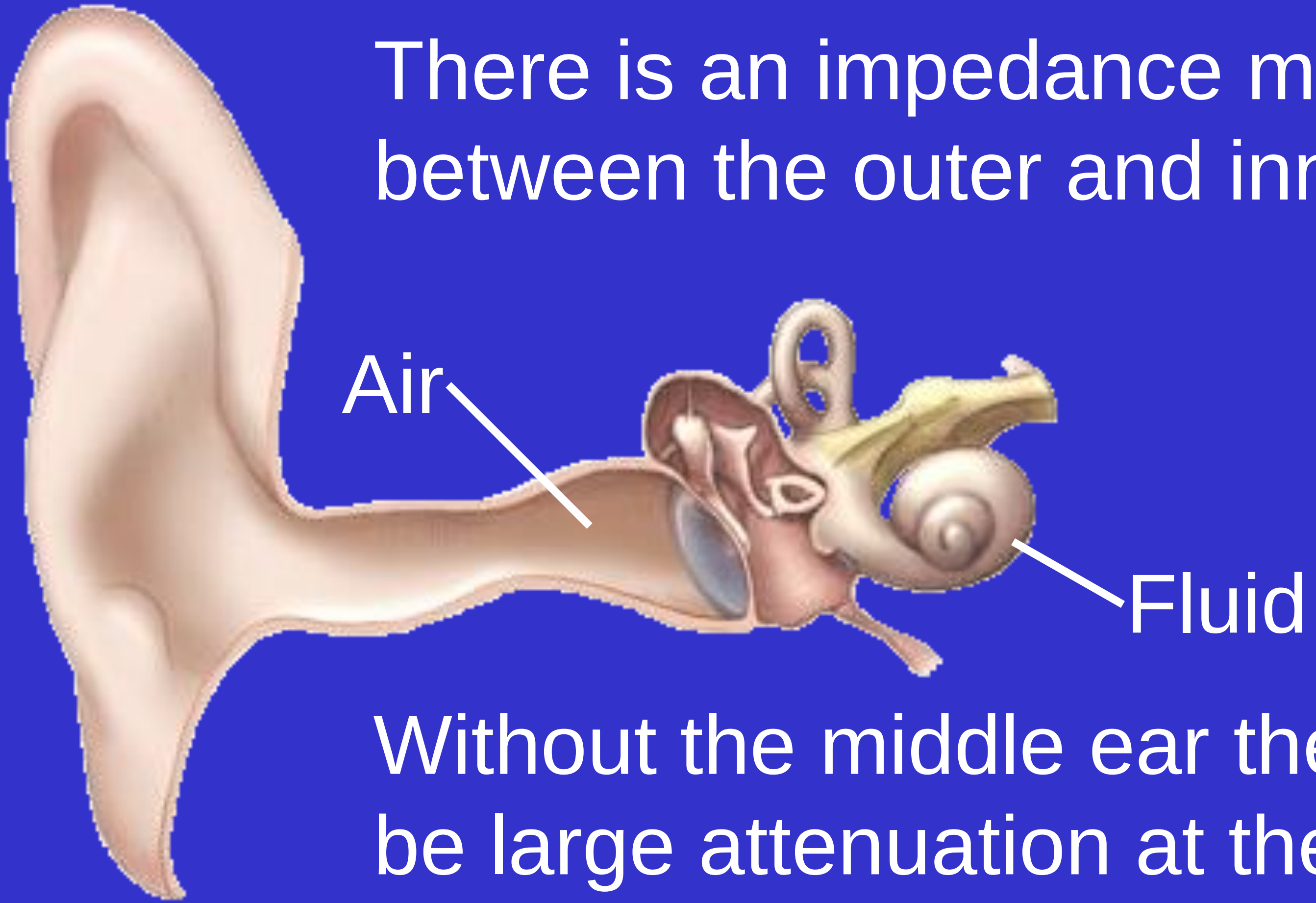
W. J. Mullin, W. J. George, J. P. Mestre, and S. L. Velleman, *Fundamentals of sound with applications to speech and hearing* (Allyn and Bacon, Boston, 2003)

Outline

I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
    a. Anatomy
    b. Theories of cochlear function
    c. Traveling wave theory
VII. Current work
    a. Physical model
    b. Mathematical model

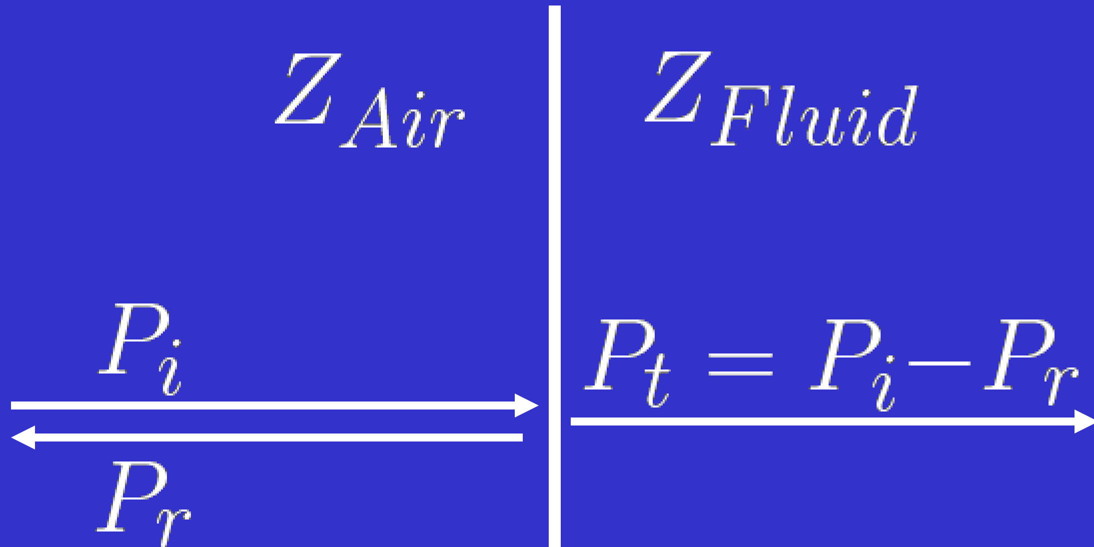
The middle ear

There is an impedance mismatch between the outer and inner ears



Without the middle ear there would be large attenuation at the air-fluid boundary

Transmission and reflection



Reflection Coefficient

$$R = \frac{P_r}{P_i}$$

Transmission Coefficient

$$T = \frac{P_t}{P_i}$$

D. T. Blackstock, *Fundamentals of Physical Acoustics* (Wiley, New York, 2000)

Power transmission

Doing some math gives the power transmission coefficient

$$\tau = \frac{4Z_{Air}Z_{Fluid}}{(Z_{Fluid}+Z_{Air})^2}$$

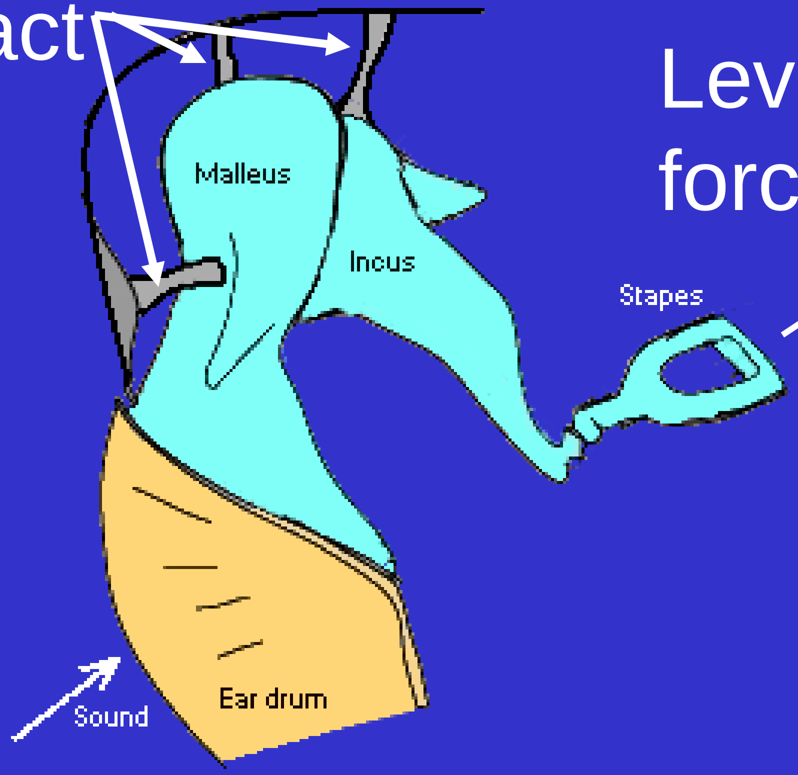
Plugging in numbers gives the attenuation

$$\tau = 1\times10^{-3} \rightarrow -30dB$$

W. J. Mullin, W. J. George, J. P. Mestre, and S. L. Velleman, *Fundamentals of sound with applications to speech and hearing* (Allyn and Bacon, Boston, 2003)

Ossicles as levers

Ligaments act as fulcrums

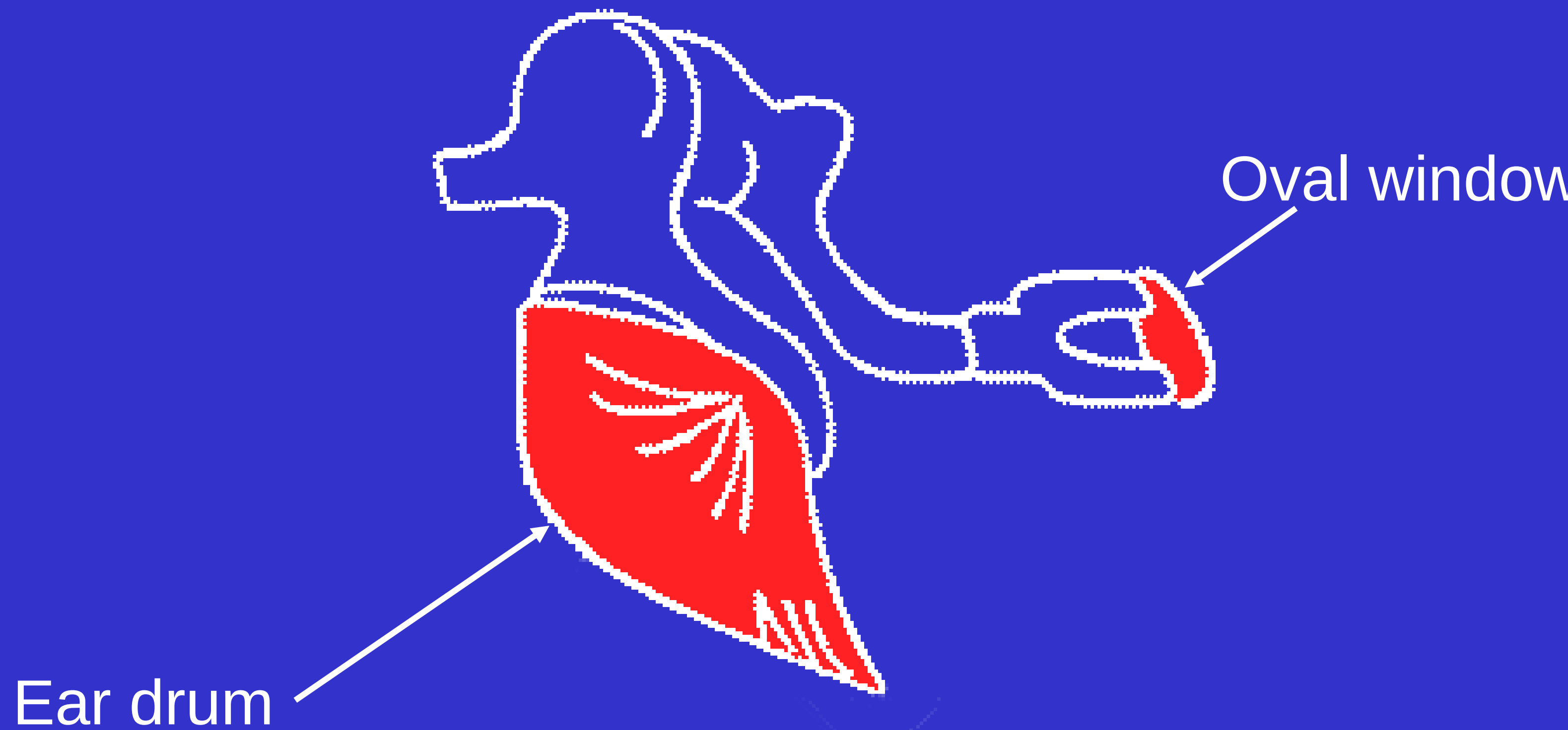


Levers increase force by 30%

$$F_{Oval} = 1.3 \times F_{Drum}$$

http://hyperphysics.phy-astr.gsu.edu/hbase/sound/imgsou/oss3.gif

Stapes footprint



$$A_{Oval} = \frac{1}{19} A_{Drum}$$

http://www.ssc.education.ed.ac.uk/courses/pictures/hearing8.gif

Impedance match

$$P_{Oval} = \frac{F_{Oval}}{A_{Oval}} = 1.3 \times 19 \frac{F_{Drum}}{A_{Drum}} = 25 P_{Drum}$$

Since $I \propto P^2$

Sound intensity

the sound intensity increases 625 times, or $28\ dB$

Outline

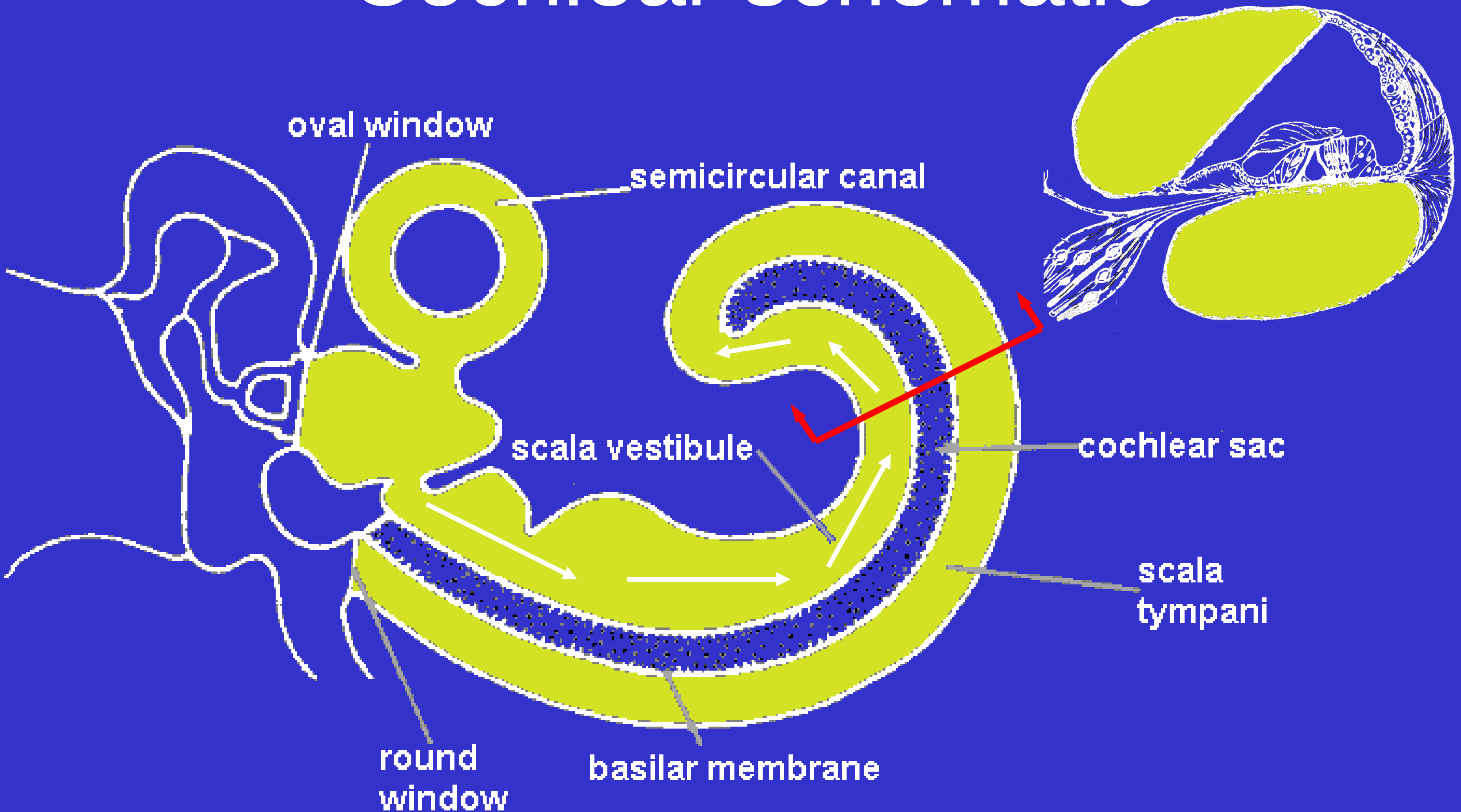
I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
    a. Anatomy
    b. Theories of cochlear function
    c. Traveling wave theory
VII. Current work
    a. Physical model
    b. Mathematical model

The inner ear

The cochlea transduces sound into electrochemical signals

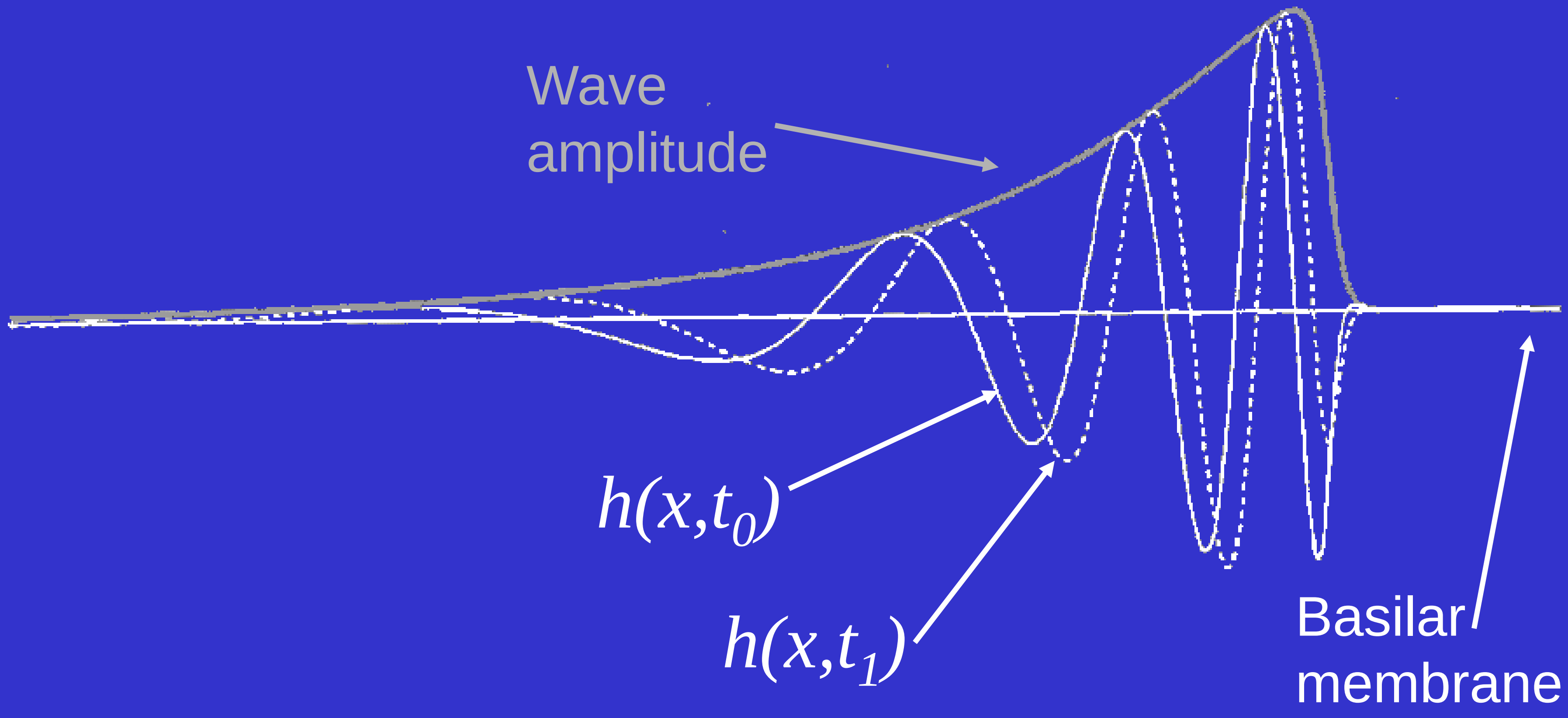
The cochlea is a Fourier analyzer that separates frequency information for the brain

Cochlear schematic



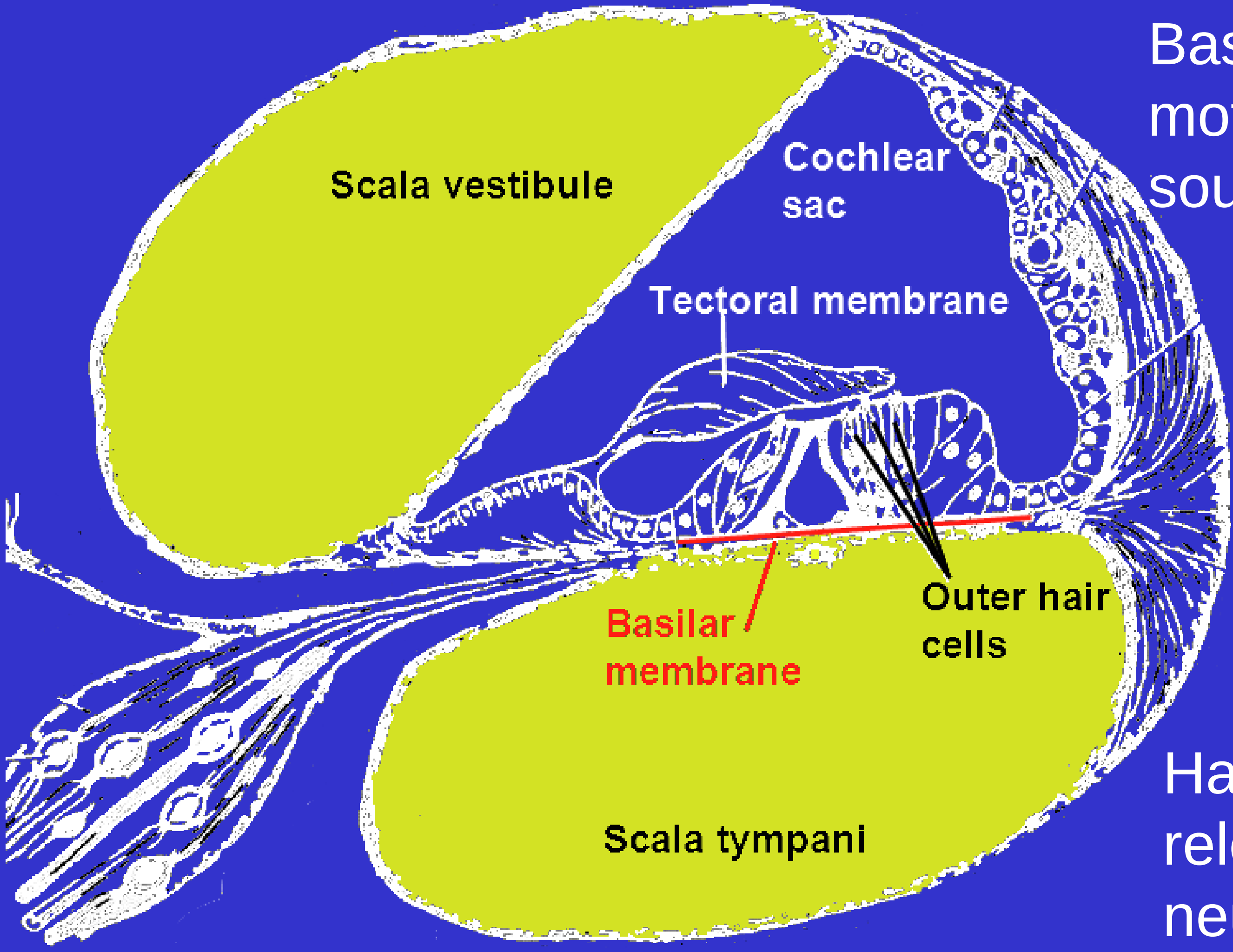
W. Yost, *Fundamentals of Hearing* (Academic Press, San Diego, 2000)

Waves on the basilar membrane



W. R. Zemiln, *Speech and Hearing Science* (Allyn and Bacon, Boston, 1998)

Cochlear cross-section

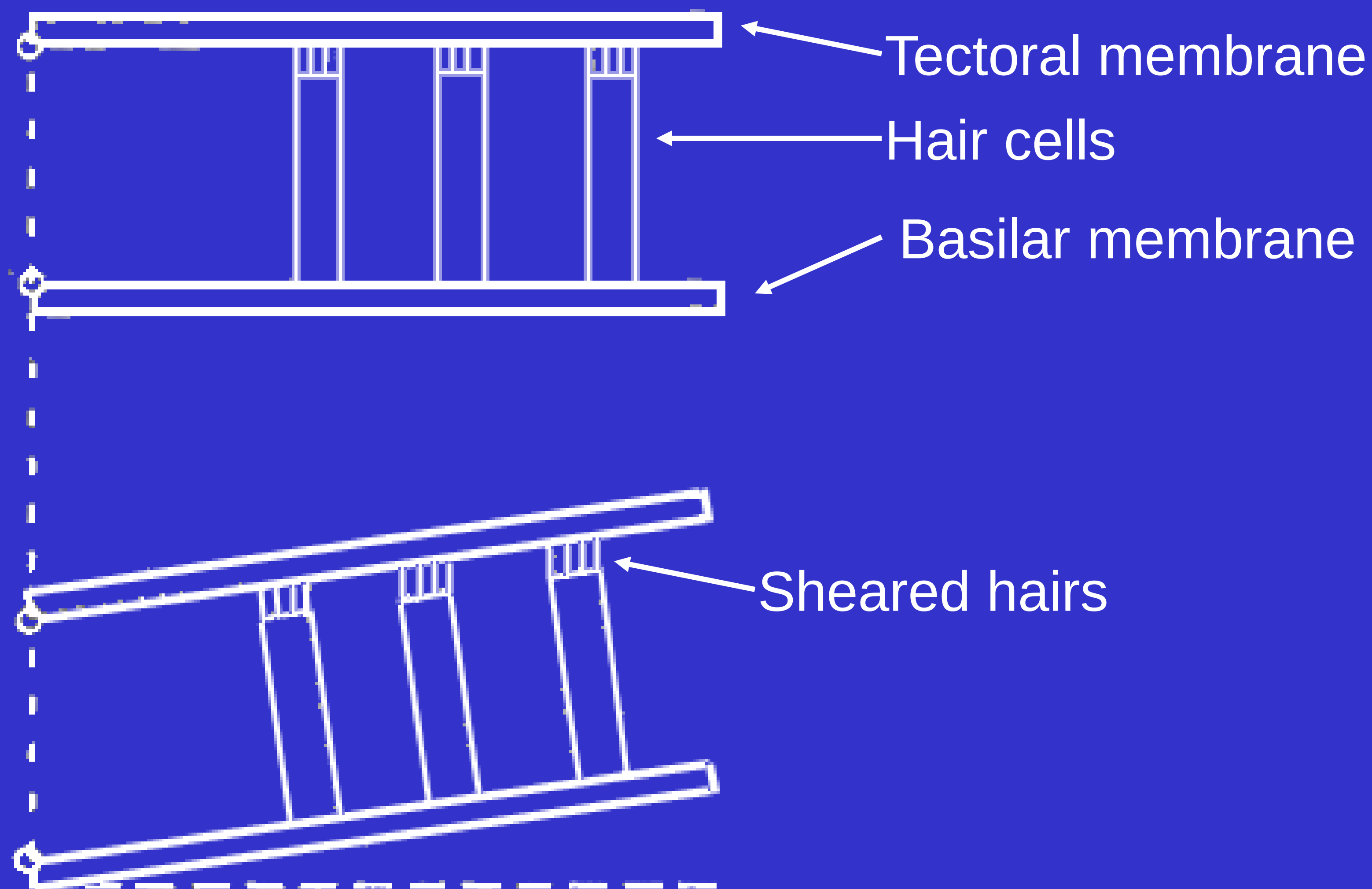


Basilar membrane motion transduces sound signal

Hair cell shearing releases neurotransmitters

W. R. Zemiln, *Speech and Hearing Science* (Allyn and Bacon, Boston, 1998)

Hair cell shearing



W. R. Zemiln, *Speech and Hearing Science* (Allyn and Bacon, Boston, 1998)

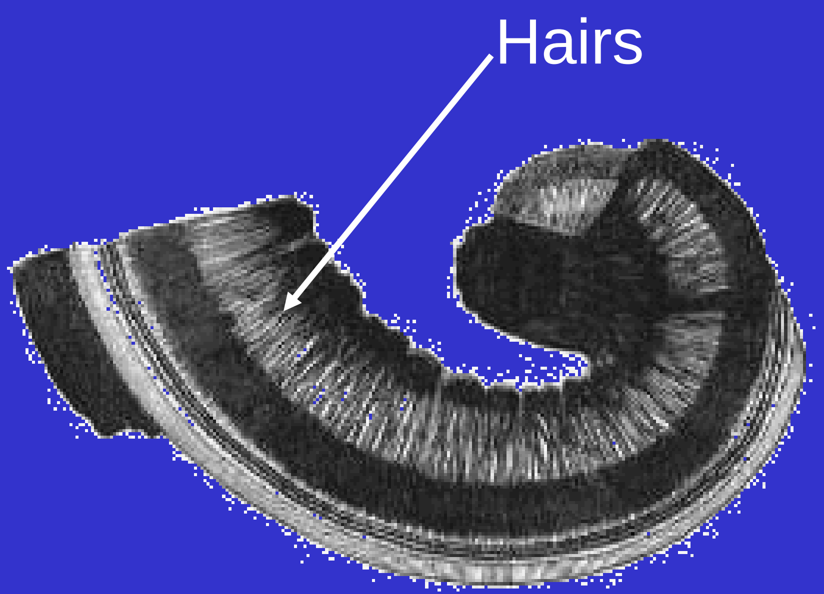
Outline

I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
   a. Anatomy
   b. Theories of cochlear function
   c. Traveling wave theory
VII. Current work
   a. Physical model
   b. Mathematical model

Helmholtz's resonance theory (1857)

One hair cell = one resonant frequency

The sum signal of the vibrating hairs reproduces the sound



Fourier analysis

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{\imath \omega t} \mathrm{d}t$$

http://www.vimm.it/cochlea/cochleapages/overview/helmholtz/helm.htm

H. von Helmholtz, *On the sensations of tone as a physiological basis for the theory of music*, (translated by A. J. Ellis) (Longmans, Green, and Co., London, 1895)

Problems with resonance theory

Needed 6,000+ hair cells to explain human hearing

Only ~3,000 hair cells were found

Later revisions used coupled membrane fibers as resonators

Cochlear anatomy was not well known

E. G. Wever, *Theory of Hearing* (Wiley, New York, 1949)

Competing theories

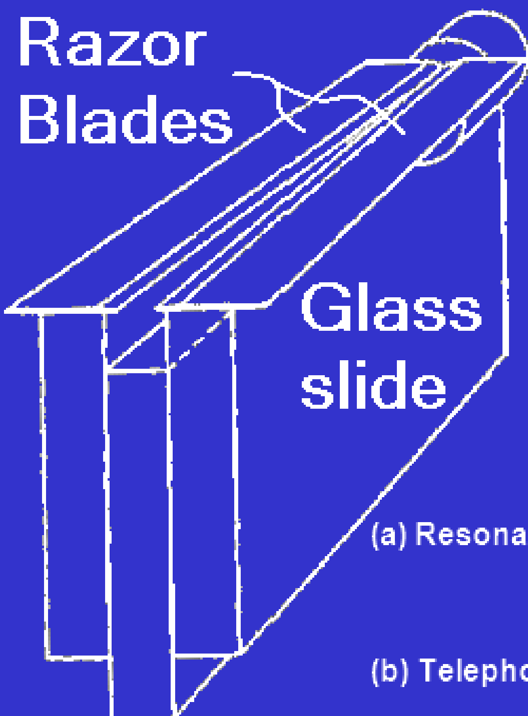
Telephone theory required each hair cell to reproduce all frequencies

Standing wave theory had hair cells detecting patterns on the basilar membrane

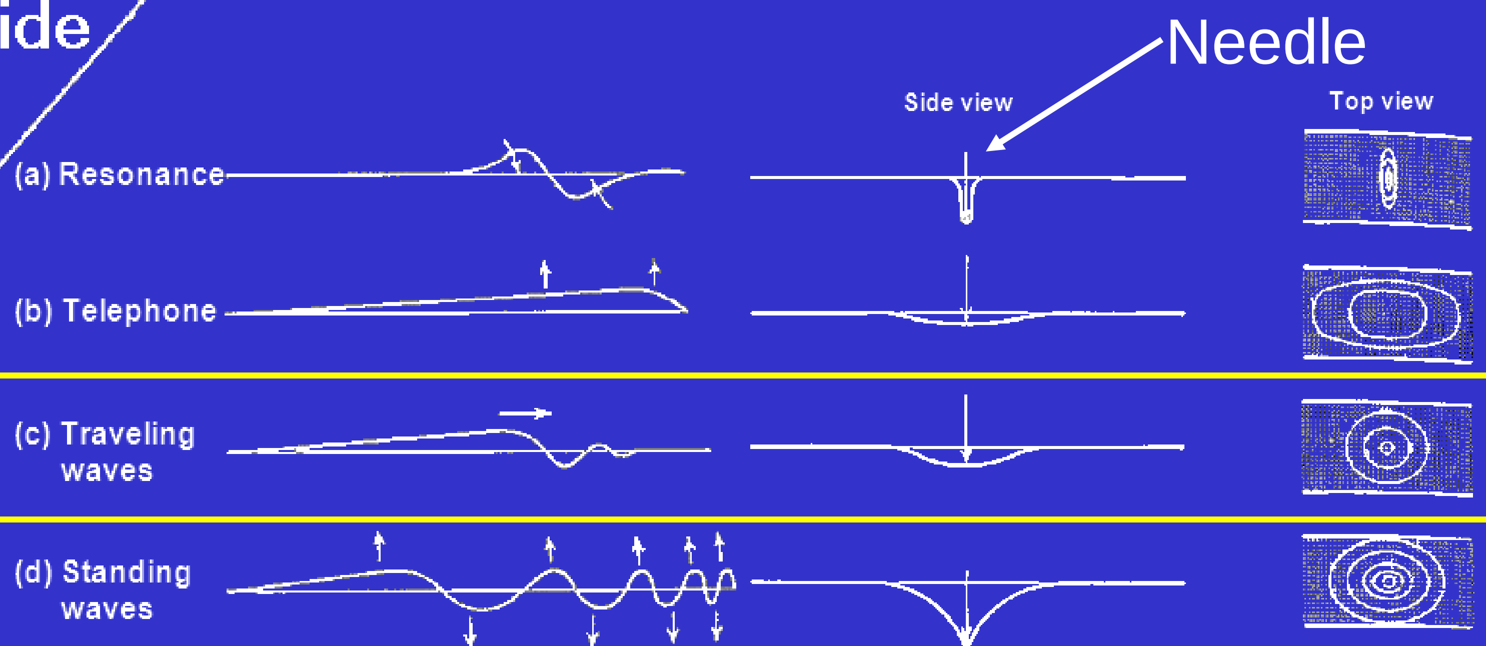
Traveling wave theory described hair cells as detecting the amplitude of a wave traveling along the basilar membrane

E. G. Wever, *Theory of Hearing* (Wiley, New York, 1949)

von Bèkèsy's physical model

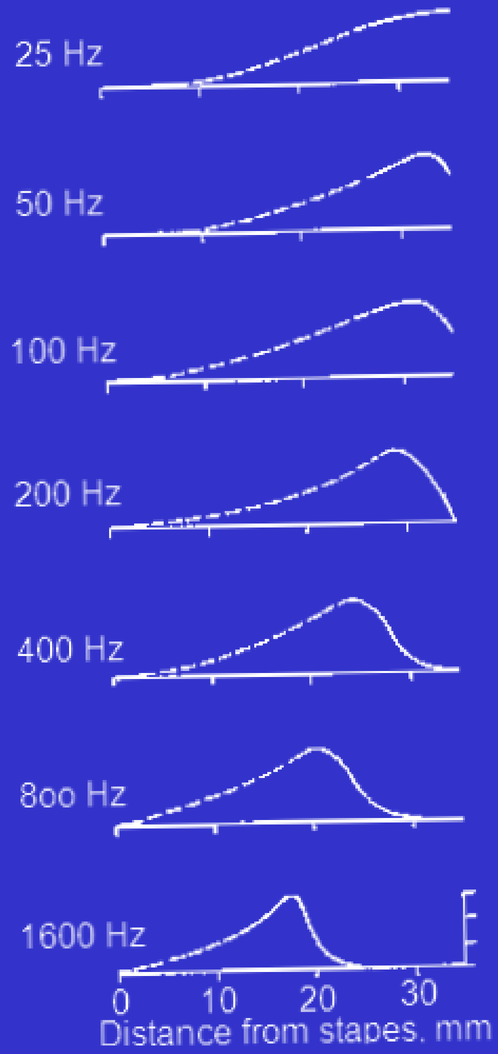


The model mimicked each of the theories



G. von Bèkèsy, *Experiments in Hearing* (McGraw-Hill, New York, 1960)

von Bèkèsy's observations



Saw traveling waves in mammalian cochleae

Position of peak depends on frequency

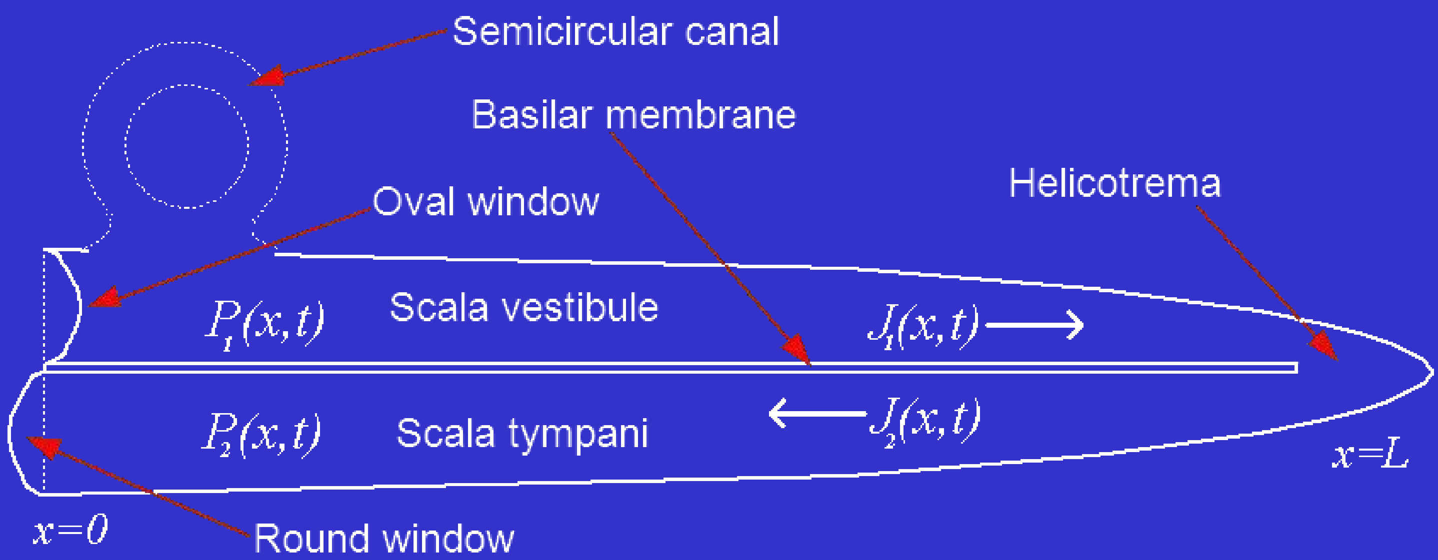
Spatial frequency separation

G. von Bèkèsy, *Experiments in Hearing* (McGraw-Hill, New York, 1960)

Outline

I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
   a. Anatomy
   b. Theories of cochlear function
   c. Traveling wave theory
VII. Current work
   a. Physical model
   b. Mathematical model

Traveling wave description



$$p = P_1 - P_2$$ relative pressure

$$j = J_1 - J_2$$ relative current; induced by pressure gradients

Pressure-current relation

Changing the relative current in time changes the relative pressure in space

$$\rho \frac{\partial}{\partial t} j = -bl \frac{\partial}{\partial x} p$$

fluid viscosity

fluid density

scala height

Incompressibility

Assuming the fluid is incompressible means that a change in relative current gives a change in basilar membrane displacement $h(x,t)$

$$2b\frac{\partial}{\partial t}h + \frac{\partial}{\partial x}j = 0$$

basilar membrane displacement

The wave equation

Stiffness relates pressure and displacement

$$p(x,t) = K(x)h(x,t)$$

Plug this in to get the wave equation

$$\frac{\partial^2}{\partial t^2}h(x,t) = \frac{l}{2\rho}\frac{\partial^2}{\partial x^2}K(x)h(x,t)$$

Stiffness

The stiffness of the basilar membrane is described by

$$K(x) = \alpha e^{-\beta x}$$

$$K(0) = \alpha \qquad K(L) = \alpha e^{-\beta L}$$

Plug this in to get

$$\frac{\partial^2}{\partial t^2} h(x,t) = \frac{\alpha l}{2\rho} \frac{\partial^2}{\partial x^2} e^{-\beta x} h(x,t)$$

G. Ehret, *J. Acoust. Soc. Am.* 64, 1723 (1978)

T. Duke and F. Jülicher, *Phys. Rev. Lett.* 90, 158101 (2003)

Solution

This equation separates into

separation constant

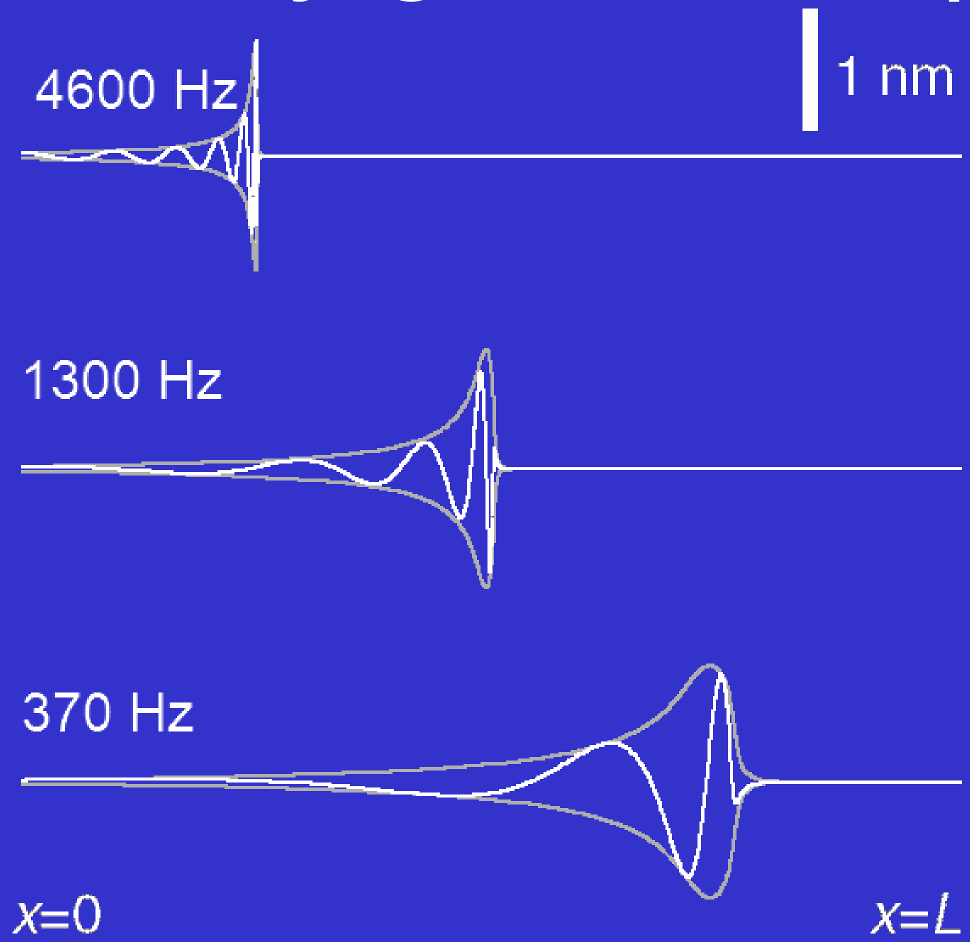
$$T(t) = B\sin(ckt)$$

$$X(x) = Ce^{\beta x} J_0\left(\frac{2\sqrt{e^{\beta x}}k}{\sqrt{\alpha}\beta}\right) + 2De^{\beta x} Y_0\left(\frac{2\sqrt{e^{\beta x}}k}{\sqrt{\alpha}\beta}\right)$$

We can go no further analytically

Usually the equation is solved numerically

Numerically generated plots

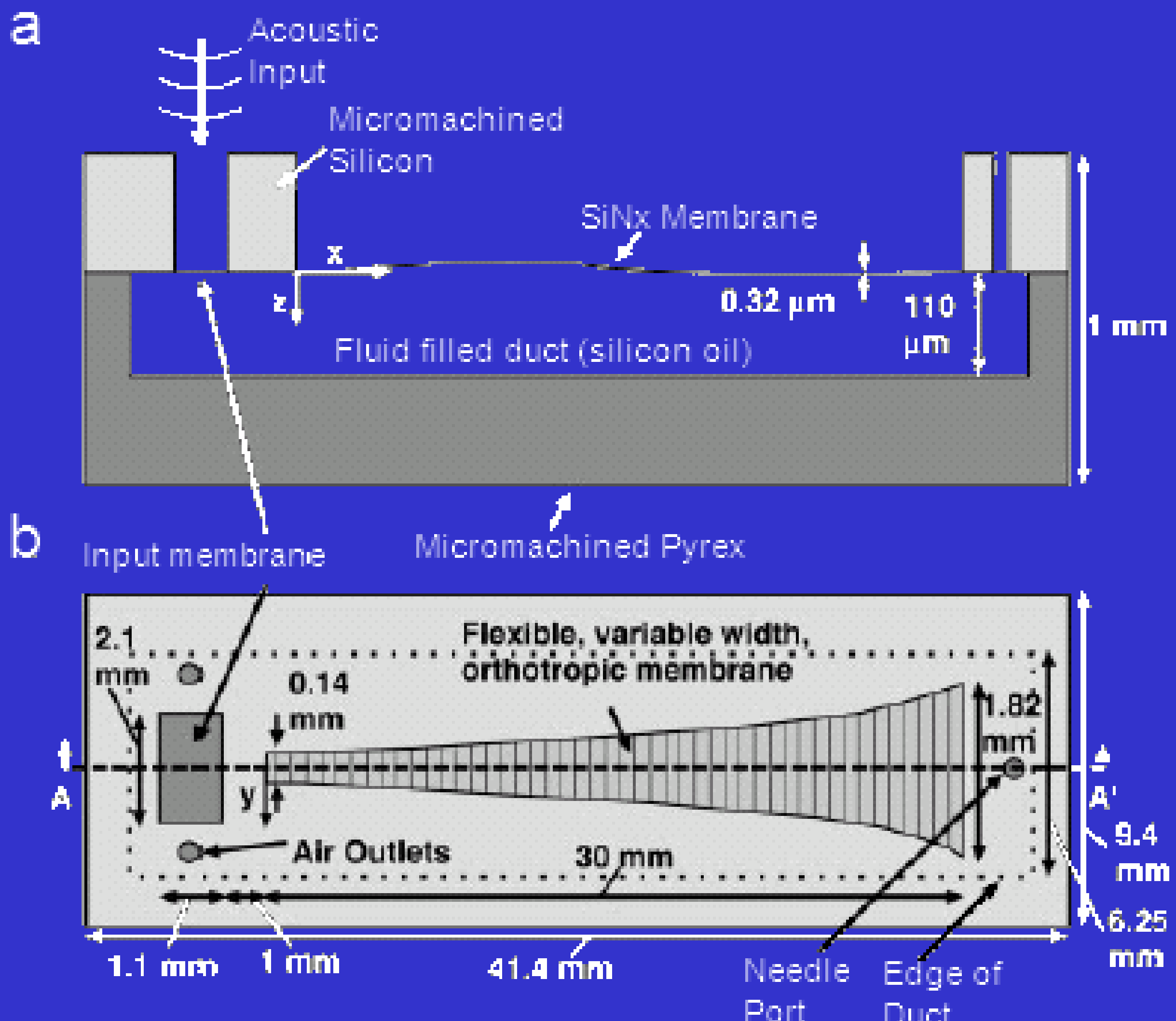


T. Duke and F. Jülicher, *Phys. Rev. Lett.* 90, 158101 (2003)

Outline

I. Ear anatomy
II. Outer ear resonator
III. Middle ear impedance matching
IV. Inner ear
    a. Anatomy
    b. Theories of cochlear function
    c. Traveling wave theory
VII. Current work
    a. Physical model
    b. Mathematical model

Recent cochlear model



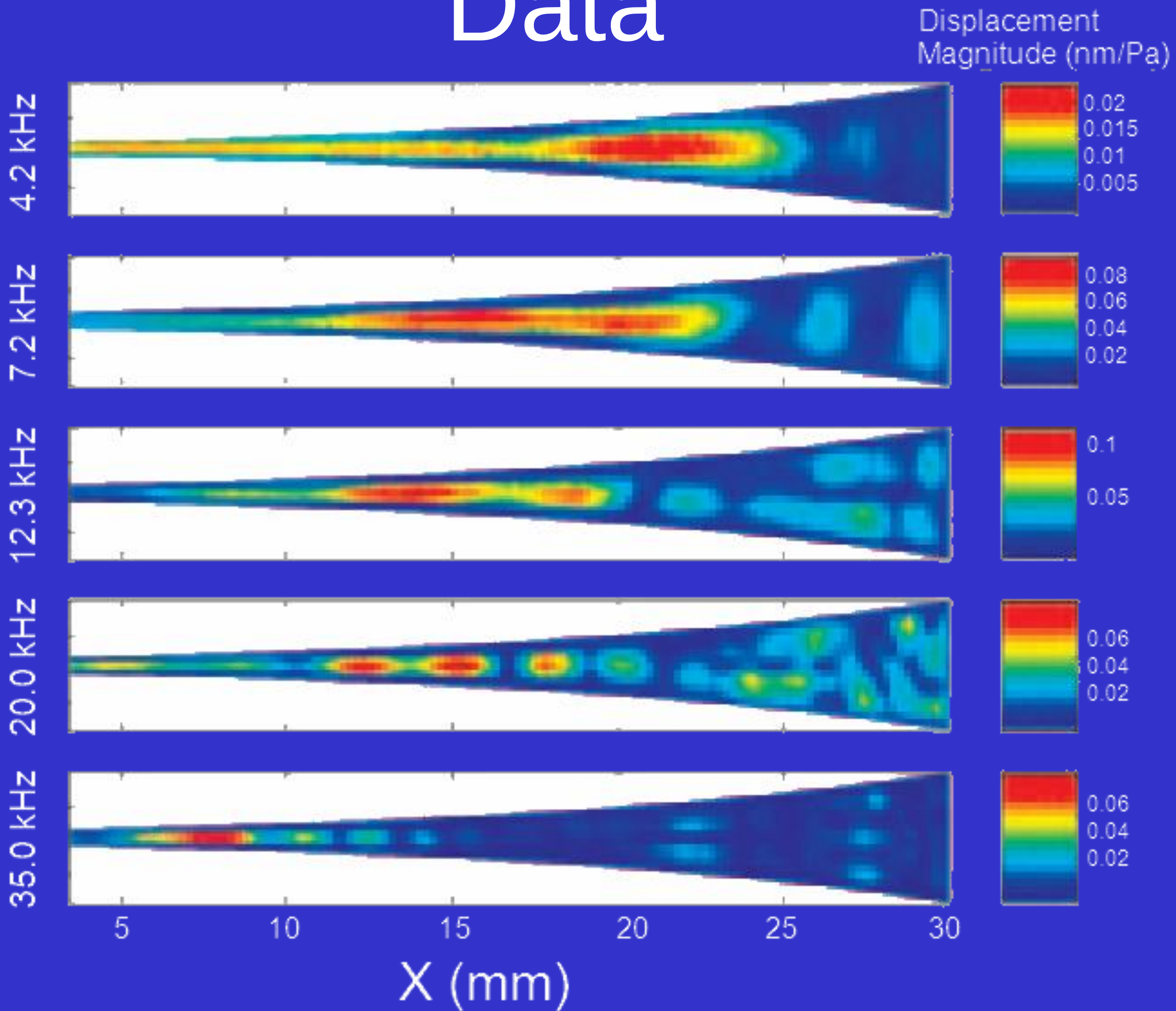
Life size

Si fabrication

Mass producible

R. D. White and K. Grosh, *PNAS* 102, 1296 (2005)

Data



R. D. White and K. Grosh, *PNAS* 102, 1296 (2005)

Outline

I. Ear anatomy

II. Outer ear resonator

III. Middle ear impedance matching

IV. Inner ear

  a. Anatomy

  b. Theories of cochlear function

  c. Traveling wave theory

VII. Current work

  a. Physical model

  b. Mathematical model

Recent math model

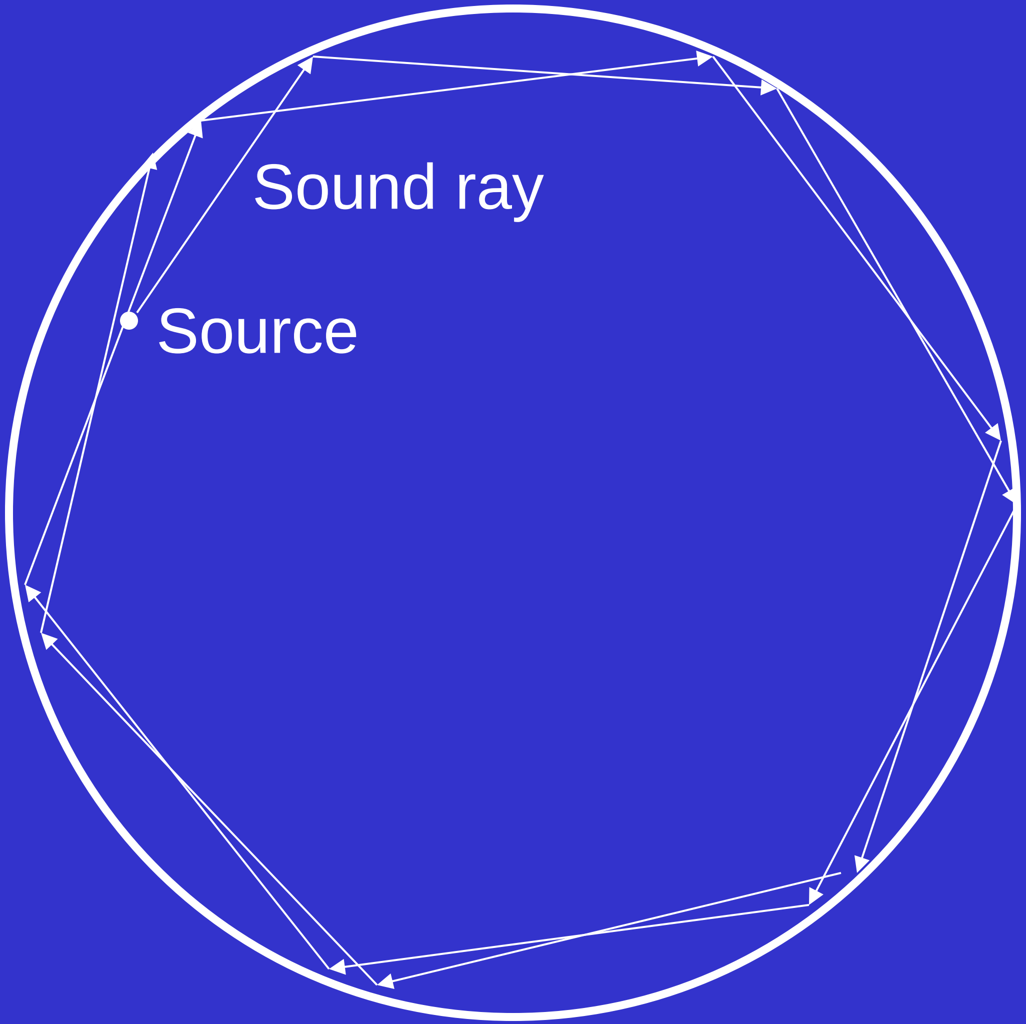
Why is the cochlea spiraled?

Previously though to have no function

Data correlate number of turns to low frequency sensitivity in various mammals

C. West, *J. Acoust. Soc. Am.* 77, 1091 (1985)
D. Manoussaki, E. K. Dimitriadis, and R. Chadwick, *Phys. Rev. Lett.* 96, 088701 (2006)

Rayleigh's whispering gallery



Sound confined to boundary

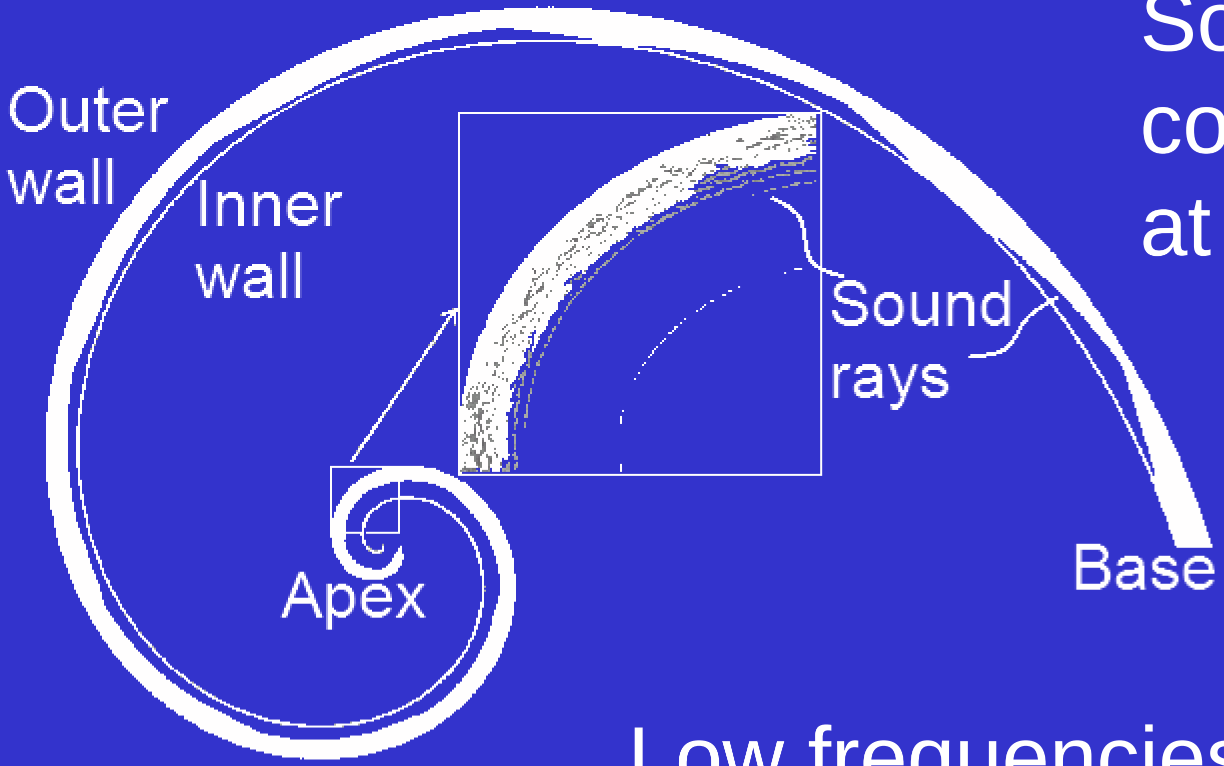
L. Rayleigh, *The Theory of Sound*, vol. II (MacMillan and Co., New York, 1895)

Spiral Boundary

a > b

http://biomed.brown.edu/Courses/BI108/2006-108websites/group10cochlearimplant/images/greenwood_function.png

Low frequency amplification



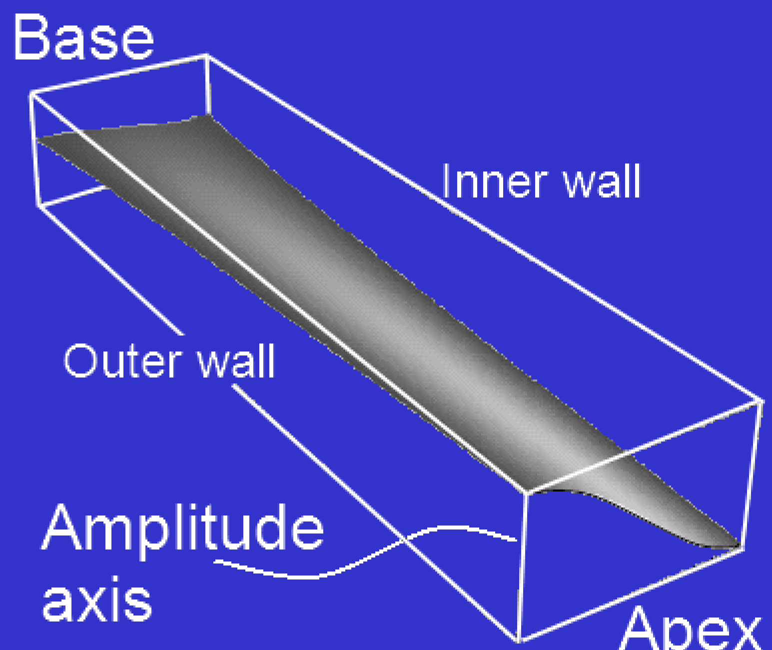
Sound concentrated at boundary

Low frequencies are transduced at the apex

D. Manoussaki, E. K. Dimitriadis, and R. Chadwick, *Phys. Rev. Lett.* 96, 088701 (2006)

Math model

Solving the traveling wave equation through a spiral cochlea shows that the sound concentrates on the outer wall of the cochlea



D. Manoussaki, E. K. Dimitriadis, and R. Chadwick, *Phys. Rev. Lett.* 96, 088701 (2006)

Conclusion

The ear uses resonance, impedance matching, and Fourier analysis

Traveling wave theory most accurately describes cochlear dynamics

Theories of cochlear function are still advancing with physical and mathematical models

Appendix A

Transmission

Transmission and reflection

Z_{Air} Z_{Fluid}

P_i

P_r

$P_t = P_i - P_r$

Reflection Coefficient

$$R = \frac{P_r}{P_i}$$

Transmission Coefficient

$$T = \frac{P_t}{P_i}$$

D. T. Blackstock, *Fundamentals of Physical Acoustics* (Wiley, New York, 2000)

Transmission and reflection

Use

$$R + T = 1, \; Z = \frac{P}{u} = \rho c, \text{ and } u_i - u_r = u_t$$

to get

$$R = \frac{Z_{Fluid} - Z_{Air}}{Z_{Fluid} + Z_{Air}}, \; T = \frac{2Z_{Fluid}}{Z_{Fluid} + Z_{Air}}$$

Power

Sound intensity is the time averaged integral of pressure times velocity

$$I = \frac{1}{t_{av}} \int_0^{t_{av}} Pu\,\mathrm{d}t \quad \rightarrow \quad I = \frac{1}{Z}\frac{1}{t_{av}} \int_0^{t_{av}} P^2\mathrm{d}t = \frac{P_{rms}^2}{Z}$$

The power transmission coefficient is then

$$\tau = \frac{I^t}{I^i} = \frac{(P_{rms}^t)^2/Z_{Fluid}}{(P_{rms}^i)^2/Z_{Air}}$$

Power transmission coefficient

Since (using the definition of T)

$$(P^t_{rms})^2 = T^2 (P^i_{rms})^2$$

the power transmission coefficient is

$$\tau = T^2 \frac{Z_{Air}}{Z_{Fluid}} = \frac{4 Z_{Air} Z_{Fluid}}{(Z_{Fluid} + Z_{Air})^2}$$

Impedance values

$$Z_{Air} = 4.15 \times 10^2 \frac{kg \cdot s}{m^2}$$

$$Z_{Fluid} = 1.44 \times 10^6 \frac{kg \cdot s}{m^2}$$

Power transmission

Doing some math gives the power transmission coefficient

$$\tau = \frac{4Z_{Air}Z_{Fluid}}{(Z_{Fluid}+Z_{Air})^2}$$

Plugging in numbers gives the attenuation

$$\tau = 1\times10^{-3} \rightarrow -30dB$$

W. J. Mullin, W. J. George, J. P. Mestre, and S. L. Velleman, *Fundamentals of sound with applications to speech and hearing* (Allyn and Bacon, Boston, 2003)

Appendix B

Spiral cochlea

Cochlear equations

Start with an irrotational fluid of velocity

$$\mathbf{v} = \nabla\Phi$$

velocity potential

and whose mass has no source or sink

$$\nabla^2\Phi = 0$$

Navier-Stokes

Use a linearized momentum equation derived from the Navier-Stokes equation for an incompressible fluid with viscosity μ and negligible body forces (like gravity)

$$-\nabla P_2 + \mu \nabla^2 \mathbf{v} = \rho \frac{\partial}{\partial t} \mathbf{v}$$

Navier-Stokes

Spatially integrate and use the mass conservation relation to get

$$P_2 + \rho\frac{\partial}{\partial t}\Phi = 0$$

Continuity

Change in the velocity potential gives a change in the membrane displacement

$$\frac{\partial}{\partial t} h = \frac{\partial}{\partial z} \Phi$$

The equation of motion for the membrane is

stiffness

mass

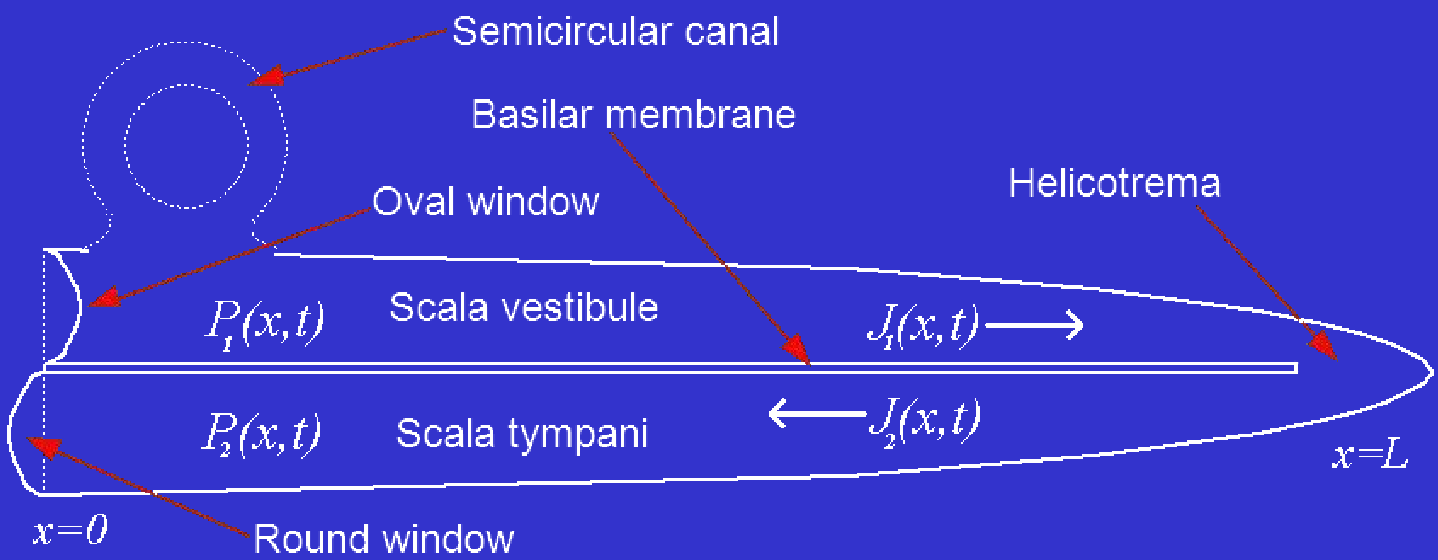
$$m \frac{\partial^2}{\partial t^2} h + \beta \frac{\partial}{\partial t} h + \kappa h = -p$$

damping

Appendix C

Traveling wave derivation

Traveling wave description



$$p = P_1 - P_2$$ relative pressure

$$j = J_1 - J_2$$ relative current

Pressure-current relation

Changing the relative current in time
changes the relative pressure in space

fluid viscosity

$$\rho \frac{\partial}{\partial t} j = -bl \frac{\partial}{\partial x} p$$

fluid density

scala height

Incompressibility

Assuming the fluid is incompressible means that a change in relative current gives a change in basilar membrane displacement $h(x,t)$

$$2b\frac{\partial}{\partial t}h+\frac{\partial}{\partial x}j = 0$$

basilar membrane displacement

Time derivative

Take the time derivative

$$2b\frac{\partial^2}{\partial t^2}h+\frac{\partial}{\partial x}\frac{\partial}{\partial t}j = 0$$

Plug in the pressure current relation

$$2\rho b\frac{\partial^2}{\partial t^2}h = \frac{\partial}{\partial x}\left(bl\frac{\partial}{\partial x}p\right)$$

The wave equation

Stiffness relates pressure and displacement

$$p(x,t) = K(x)h(x,t)$$

Plug this in to get the wave equation

$$\frac{\partial^2}{\partial t^2}h(x,t) = \frac{l}{2\rho}\frac{\partial^2}{\partial x^2}K(x)h(x,t)$$

Stiffness

The stiffness of the basilar membrane is described by

$$K(x) = \alpha e^{-\beta x}$$

$K(0) = \alpha$ $K(L) = \alpha e^{-\beta L}$

Plug this in to get

$$\frac{\partial^2}{\partial t^2} h(x,t) = \frac{\alpha l}{2\rho} \frac{\partial^2}{\partial x^2} e^{-\beta x} h(x,t)$$

G. Ehret, *J. Acoust. Soc. Am.* 64, 1723 (1978)
T. Duke and F. Jülicher, *Phys. Rev. Lett.* 90, 158101 (2003)

Solution

This equation separates into

separation constant

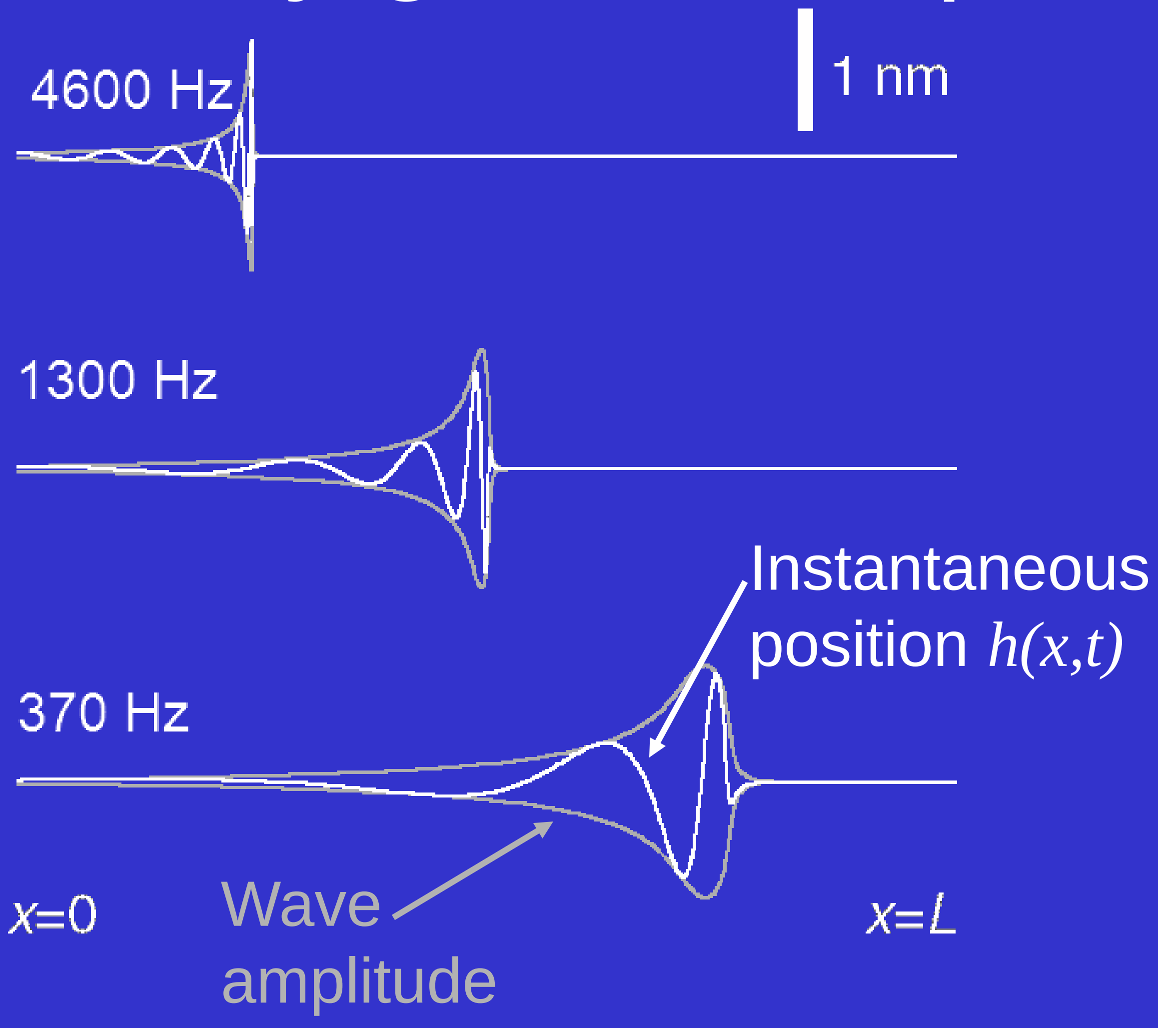
$$T(t) = B\sin(ckt)$$

$$X(x) = Ce^{\beta x}J_0\left(\frac{2\sqrt{e^{\beta x}}k}{\sqrt{\alpha}\beta}\right) + 2De^{\beta x}Y_0\left(\frac{2\sqrt{e^{\beta x}}k}{\sqrt{\alpha}\beta}\right)$$

We can go no further analytically

Usually the equation is solved numerically

Numerically generated plots



T. Duke and F. Jülicher, *Phys. Rev. Lett.* 90, 158101 (2003)